Actividad EDO.

Ecuaciones Exactas.

5. $(\tan(y)-2)\,dx + \left(x\sec^2(y) - \frac{1}{y}\right)dy = 0$

$M(x,y) = \tan(y)-2 \qquad N(x,y) = x\sec^2(y) - \frac{1}{y}$

Comprobando si son exactas

$\frac{\partial M}{\partial y} = \sec^2 y - 0 = \sec^2(y) \qquad \frac{\partial N}{\partial x} = \sec^2(y)\cdot(1) - 0 = \sec^2(y)$

Iguales $\Rightarrow$ Exactas

Integrando

$\int N(x,y = \int x\sec^2(y) + \frac{1}{y}\,dy = \overbrace{x\tan(y) + \ln(y) + g(x)}^{*}$

Derivando

$M(x,y) = \frac{d}{dx}\left(x\tan(y) + \cancel{\ln(y)}^{0} + g(x)\right)$

$\cancel{\tan(y)}-2 = \cancel{\tan(y)} + g'(x) \Rightarrow -2 = g'(x)$

Integrando

$\int -2\,dx = \int g'(x)\,dx$

$-2x + C = g(x)$

Sustituyendo $g(x)$ en $*$

$x\tan(y) + \ln(y) - 2x + C = cte$

$\therefore\; x\tan(y) + \ln(y) - 2x = C$ // Solucion General Implicita

6. $\left(\frac{2x}{y} - \frac{3y^2}{x^4}\right)dx + \left(\frac{2y}{x^3} - \frac{x^2}{y^2} + \frac{1}{\sqrt{y}}\right)dy = 0$

$M(x,y) = \frac{2x}{y} - \frac{3y^2}{x^4}$ $\qquad$ $N(x,y) = \frac{2y}{x^3} - \frac{x^2}{y^2} + \frac{1}{\sqrt{y}}$

Comprobando si son exactas

$\frac{\partial M}{\partial y} = -\frac{2x}{y^2} - \frac{6y}{x^4}$; $\frac{\partial N}{\partial x} = -\frac{6y}{x^4} - \frac{2x}{y^2} + 0.$ $\qquad$ $y^{-1/2 + 2/2} = y^{1/2} = \sqrt{y}$

Iguales

$\downarrow$

Exactas

Integrando $\int N(x,y)\,dy = \int \frac{2y}{x^3} - \frac{x^2}{y^2} + \frac{1}{\sqrt{y}}\,dy = \overbrace{\frac{y^2}{x^3} + \frac{x^2}{y} + \sqrt{y}} + g(x).$

Derivando

$* \quad M(x,y) = \frac{d}{dx}\left(\frac{y^2}{x^3} + \frac{x^2}{y} + \sqrt{y} + g(x)\right) = -\frac{3y^2}{x^4} + \frac{2x}{y} + 0 + g'(x)$

$\cancel{\frac{2x}{y}} - \cancel{\frac{3y^2}{x^4}} = -\cancel{\frac{3y^2}{x^4}} + \cancel{\frac{2x}{y}} + g'(x) \Rightarrow g'(x) = 0.$

Integrando

$\int g'(x)\,dx = \int 0\,dx$

$g(x) = 0$

Sustituyendo $g(x)$ en $*$ $\quad \frac{y^2}{x^3} + \frac{x^2}{y} + \sqrt{y} + 0 = cte.$

$\therefore \frac{y^2}{x^3} + \frac{x^2}{y} + \sqrt{y} = C$ // Sol. Gral Implícita